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ROLE OF α_1 AND α_2 NEAR RING IN BOOLEAN S-NEAR RING

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ABSTRACT : -

In this paper we have proved some results on Boolean S-near ring using the concepts of regular near ring, idempotents, left cancellation law etc. It is proved that N is a S-near ring iff N is boolean whenever N is regular. Every Boolean S-near ring is both α_1 and α_2 near ring with the converse in the case of α_2 near ring. Also, as a characterization theorem it is proved that a Boolean regular near ring is an S-near ring in each of the following cases (i) N is an IFP with identity (ii) $Na = aNa$ for all $a \in N$ (iii) N is subcommutative.

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Keywords :-

α_1 near ring, α_2 near ring, strongly regular, subcommutativity, S_1 near ring, S_2 near ring,.

INTRODUCTION

Near rings can be thought of as generalized rings : if in a ring we ignore the commutativity of addition and one distributive law, we get a near ring. Gunter Pilz [2] "Near rings" is an extensive collection of the work done in the area of near rings.

A **near ring** N is a system $(N,+,\cdot)$ such that $(N,+)$ is a group (not necessarily abelian), $(N,\cdot)$ is a semigroup, the right distributive law holds, i.e. $(x+y)z = xz +$

yz for each x, y, z in N ; and $x \cdot 0 = 0$ for every x in N [6]. A near ring N is an $\boldsymbol{S}$**-near ring** if $a \in Na$ for each $a \in N$ [6]. Let N be a right near ring, if (i) for every a in N there exists x in N such that $a = xax$ then we say N is an $\boldsymbol{\alpha_1}$ **near ring**. (ii) for every a in N^* there exists x in N^* such that $x = xax$ then we say N is $\boldsymbol{\alpha_2}$ **near ring** [27].

Preliminaries

Definition 2.1 [4]

The near rings N are **boolean** if $x^2 = x$ for each $x \in N$.

Definition 2.2 [6]

A near ring N is defined to be **left bipotent** if $Na = Na^2$ for each a in N.

Definition 2.3 [6] A near ring N is **regular** if for each a in N, there exists x in N such that $a = axa$.

Definition 2.4 [3]

If all non zero elements of N are left(right) cancellable then we say that N fulfills the left(right) cancellation law.

Notation 2.5 [25]

E denotes the set of all idempotent of N ($a \in E$ iff $a^2 = a$).

Definition 2.6[3]

N is said to fulfill the **Insertion of Factors Property (IFP)** provided that for all a, b, n in N, $ab = 0 \Rightarrow anb = 0$.

Definition 2.7 [1]

N is called a $\boldsymbol{P_k}$ **near ring (**$\boldsymbol{P_k'}$ **near ring)** if there exists a positive integer k such that $x^k N = xNx$ $(Nx^k = xNx)$ for all $x \in N$.

Definition 2.8 [8]

N is said to be **subcommutative** if $Na = aN$ for all $a \in N$.

Notation 2.9 [25]

N^* denotes the set of all nonzero elements of N, i.e.,$N^* = N - \{0\}$.

Definition 2.10 [25]

N is called an $\boldsymbol{S_1}$ **near ring (**$\boldsymbol{S_2}$ **near ring)** if for every $a \in N$, there exists $x \in N^*$ such that $axa = xa$ $(axa = ax)$.

Lemma 2.11 [4]

If N is a boolean near ring, then $xy = xyx$ for each $x, y \in N$.

Definition 2.12 [2]

A near ring N is said to be **strongly regular** if for each $a \in N$, there exists an element $x \in N$ such that $a = xa^2$.

Main Results

Theorem 3.1

Let N be a reduced near ring. N is left bipotent iff N is boolean.

Proof:

Let N be left bipotent. Then $Na = Na^2$ for each a in N. $\Longrightarrow xa = xa^2$ for all x in N. $\Longrightarrow xa - xa^2 = 0$. $\Longrightarrow x(a - a^2) = 0$. $\Longrightarrow a - a^2 = 0$, since N is reduced. This gives $a = a^2$. Hence N is boolean. Converse follows.

Theorem 3.2

Let N be a regular near ring. N is S-near ring iff N is boolean.

Proof:

Let N be S-near ring. Then $a \in Na$ for all $a \in N$.This implies $a = xa$ for some $x \in N$. Since N is regular, for each $a \in N$, there exists $x \in N$ such that $a = axa$.This gives $a = a \cdot a = a^2$. Therefore $a = a^2$. Hence N is boolean. Conversely, let N be regular. Then for each $a \in N$, there exists $x \in N$ such that $a = axa$. Since N is boolean, $a^2 = a$. This gives $a^2 = axa$. By left cancellation law, $a = xa$. Therefore $a \in Na$. Hence N is S-near ring.

Theorem 3.3

Let N be boolean near ring. If N is S-near ring then N is regular.

Proof:

Let N be S-near ring. Then $a \in Na$ for all $a \in N$. This implies $a = xa$ for some x in N. Since N is boolean, $a = a^2 = a \cdot a = axa$. Therefore $a = axa$. Hence N is regular.

Theorem 3.4

Let N be S-near ring. If $xa = 0$ then $ax = 0$ for all $a \in N$ and for some $x \in N$.

Proof:

Let N be S-near ring. Then $a \in Na$ for all $a \in N$. This implies $a = xa$ for some $x \in N$. Now $ax = xax = 0x = 0$. Hence $ax = 0$.

Theorem 3.5

Let N be S-near ring. If N is boolean, then (i) $ax \in E$ (ii) If the left cancellation law is valid in N then $xa \in E$ for all $a \in N$ and for some $x \in N$.

Proof:

Let N be S-near ring. Then $a \in Na$ for all $a \in N$. This implies $a = xa$ for some x in N. Let N be boolean. Then $a^2 = a$ for all $a \in N$.(i) $(ax)^2 = (ax)(ax) = aax = a^2x$ (Since N is boolean). That is $(ax)^2 = ax$ and hence $ax \in E$. (ii) Consider $a(xa)^2 = a(xa)(xa) = aa(xa) = a^2(xa) = axa$ (Since N is boolean). Therefore $a(xa)^2 = axa$. Since the left cancellation is valid in N, $(xa)^2 = xa$. Thus $xa \in E$.

Theorem 3.6

Let N be subcommutative and S-near ring. If N is boolean then N is strongly regular.

Proof:

Let N be S-near ring. Then $a \in Na$ for all $a \in N$. This implies $a = xa$ for some x in N. Since N is subcommutative, $Na = aN$. Therefore for any $x \in N$, there exists $y \in N$ such that $xa = ay$. This implies $a = ay$. Now $ay = xa$. $\Longrightarrow aya = xaa = xa^2 \Longrightarrow$

$aa = xa^2 \Longrightarrow a^2 = xa^2 \Longrightarrow a = xa^2$ (Since N is boolean). Hence N is strongly regular.

Theorem 3.7

Let N be S-near ring. If N is strongly regular then N is boolean.

Proof:

Let N be S-near ring. Then $a \in Na$ for all $a \in N$. This implies $a = xa$ for some $x \in N$. Since N is strongly regular, for each $a \in N$, there exists an element $x \in N$ such that $a = xa^2$. This implies $a = xaa = a^2$. Therefore $a = a^2$. Hence N is boolean.

Theorem 3.8

Let N be S-near ring. If N is boolean, then N is α_1 near ring.

Proof:

Let N be S-near ring. Then $a \in Na$ for all $a \in N$. This implies $a = xa$ for some x in N. Since N is boolean, by lemma 2.1 we have $xa = xax$ for each $x, a \in N$. This implies $a = xax$. Hence N is α_1 near ring.

Theorem 3.9

Let N be S-near ring. N is boolean iff N is α_2 near ring.

Proof:

Let N be S-near ring. Then $x \in Nx$ for all $x \in N$. This implies $x = ax$ for some a in N. Since N is boolean, $x = x^2 = xax$. Therefore $x = xax$. In particular $x = xax$ for any $x, a \in N^*$. Hence N is α_2 near ring. Conversely, since N is α_2 near ring, for every a in N^* there exists x in N^* such that $x = xax$. This implies $x = xx = x^2$. Therefore $x = x^2$. Hence N is boolean.

Theorem 3.10

Let N be boolean near ring. If N is commutative then N is S_1 near ring.

Proof:

Since N is boolean, by lemma 2.1 we have $ax = axa$ for each $a, x \in N$ which gives $xa = axa$, since N is commutative. In particular, $xa = axa$ for any $x \in N^*$. Hence N is S_1 near ring.

Theorem 3.11

Let N be S-near ring. N is regular iff N is a S_1 near ring.

Proof:

Let N be S-near ring. Then $a \in Na$ for all $a \in N$. This implies $a = xa$ for some x in N. Since N is regular, for each a in N, there exists x in N such that $a = axa$ which gives $axa = xa$. In particular $axa = xa$ for any $x \in N^*$. Hence N is S_1 near ring. Conversely, since N is S_1 near ring, for every $a \in N$, there exists $x \in N^*$ such that $axa = xa$ which gives $axa = a$. Hence N is regular.

Corollary 3.12

If N is boolean, then N is S_2 near ring.

Theorem 3.13

Let N be a boolean near ring. If N is regular, then each of the following statements implies that N is an S-near ring. (i) N is an IFP near ring with identity. (ii) $Na = aNa$ for all $a \in N$. (iii) N is subcommutative. (iv) N is zero symmetric.

Proof:

Since N is regular, for each $a \in N$, there exists $x \in N$ such that $a = axa$. (i) Let N be an IFP near ring with identity '1' and let $a \in N$. Since N is boolean, $a^2 = a \Longrightarrow a^2 - a = 0 \Longrightarrow (a-1)a = 0$. Since N has IFP, $(a-1)xa = 0$ for all $x \in N$. $\Longrightarrow axa - xa = 0 \Longrightarrow a - xa = 0 \Longrightarrow a = xa \Longrightarrow a \in Na$ for all $x \in N$. Hence N is an S-near ring. (ii) Since $Na = aNa$, for any $x \in N$, there exists $y \in N$ such that $xa = aya$. Now $axa = a(xa) = a(aya)$ $= a^2ya = aya = xa$. Hence $axa = xa$. This implies $a = xa$. Therefore $a \in Na$. Hence N is an S-near ring. (iii) Since N is subcommutative, $Na = aN$. Therefore for any $x \in N$, there exists $y \in N$ such that $xa = ay$. Therefore $axa = a(xa) = a(ay) = a^2y = ay$. That is $axa = ay$. This implies $a = ay$ which gives $a = xa$. Therefore $a \in Na$. Hence N is an S-near ring. (iv) Let N be zero symmetric near ring. Let $a \in N$. If $a \neq 0$, we take $x = a$. Then $axa = a^2a = xa$. This gives $a = xa$. If $a = 0$ then for any $x \in N$, $a = 0 = xa$. Hence N is S-near ring.

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