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GENERALIZATION OF W-FUZZY MAPPINGS

ANBUCHELVI.M
Associate Professor of Mathematics
V.V. Vanniaperumal College for Women, Virudhunagar
Tamil Nadu, INDIA

Corresponding author: mail: rsanbuchelvi@gmail.com

ABSTRACT

This paper aims to introduce two new classes of mappings via r-fuzzy w-closed sets in the sense of ˇ*S* ostak The class of r-fuzzy w-closed sets is nothing but the generalization of w-fuzzy closed sets. Some of it's basic properties have been analyzed by giving simple proofs and suitable examples. Mappings like r-fuzzy w-continuity and r-fuzzy closed have been introduced and some theorems based on these mappings have been investigated.

Keywords: *r-w-fuzzy closed sets, r-fuzzy continuous function, r-fuzzy w- Homeomorphisms.*

1. INTRODUCTION

Chang [1] introduced the concept of fuzzy topological space. S ostak [8] developed the structure of topology in other way as generalizations of Chang's fuzzy topology. Ramadan [6] and Chattopadhyay et al [2,3] introduced a similar definition in the name of smooth topological space. Levine [5] introduced the concept of generalized closed sets in topological space. In 2000, Sundaram et al [9] introduced $\boldsymbol{\omega}$ -closed sets. In 2004, Kim and Ko [4] developed r-generalized closed sets in fuzzy topological space. In this paper, the notion of r-fuzzy $\boldsymbol{\omega}-$ closed sets is introduced and it's basic properties have been investigated. Also, the mappings such as r-fuzzy $\boldsymbol{\omega}-$ continuous, r-fuzzy $\boldsymbol{\omega}-$ irresolute r-fuzzy $\boldsymbol{\omega}-$closed map, r-fuzzy $\boldsymbol{\omega}-$open map have been introduced and their properties are studied.

2. Preliminaries:

Definition 2.1:[3, 8] A fuzzy topology on X is a map $\tau: I^X \longrightarrow I$ which satisfies the following conditions:

(1) $\tau(\overline{0}) = \tau(\overline{1}) = 1$,

(2) $\tau(\mu_1 \wedge \mu_2) \geq \tau(\mu_1) \wedge \tau(\mu_2)$

(3) $\tau(\vee\mu_i) \geq \wedge\, \tau(\mu_i)$

The pair (X, τ) is called a fuzzy topological space. A fuzzy set α is called r-fuzzy open set (or fuzzy r-open set) if $\tau(\alpha) \geq r$ and r-fuzzy closed set (or fuzzy r-closed set) if $\tau(\overline{1} - \alpha) \geq r$.

Definition 2.2: [3] Let (X, τ) be a fuzzy topological space. For each $r \in I_0$ and for each $\mu \in I^X$, the fuzzy r-closure is defined by

cl$(\mu, r) = \wedge\{\rho \in I^X : \mu \leq \rho,\ \tau(\overline{1} - \rho) \geq r\}$ and fuzzy r-interior is defined by

int$(\mu, r) = \vee\{\rho \in I^X : \mu \geq \rho,\ \tau(\rho) \geq r\}$.

Moreover, μ is fuzzy r-closed if and only if cl$(\mu, r) = \mu$.

Definition 2.3: [7] A fuzzy set μ in a fuzzy topological space (X, τ) is said to be fuzzy r-semiopen if there exists a fuzzy r-open set α such that $\alpha \leq \mu \leq cl(\alpha, r)$ and fuzzy r-semiclosed if there exists a fuzzy r-closed set α such that $int(\alpha, r) \leq \mu \leq \alpha$.

Definition 2.4: [4] A fuzzy set μ in a fuzzy topological space (X, τ) is said to be r-fuzzy generalized closed set if cl$(\mu, r) \leq \rho$ whenever $\mu \leq \rho$ and ρ is r-fuzzy open set of X. The complement of r-fuzzy generalized closed set is r-fuzzy generalized open set.

Definition 2.5: [7] Let f: (X, τ) → (Y, ρ) be a map and $r \in I_0$. Then f is called a fuzzy r-continuous map if if $f^{-1}(\mu)$ is a fuzzy r-open set of X for each fuzzy r-open set μ of Y.

3**3. r-fuzzy ω −closed sets.**

Definition 3.1: Let (X, τ) be a fuzzy topological space. A fuzzy set $\alpha \in I^X$ is said to be r-fuzzy ω closed (in short, r-fωc) set if cl$(\alpha, r) \leq \mu$ whenever $\alpha \leq \mu$ and μ is a fuzzy r-semiopen set. The complement of r-fuzzy ω closed set is r-fuzzy ω open set.

Example 3.2: Let X=I. Define three fuzzy sets $\mu_1,\ \mu_2\ and\ \ \mu_3$ on X as follows

$$\mu_1(x) = \begin{cases} 0 & if\ 0 \leq x \leq \frac{1}{2} \\ 2x - 1 & if\ \frac{1}{2} \leq x \leq 1 \end{cases}$$

$$\mu_2(x) = \begin{cases} 1 & if\ 0 \leq x \leq \frac{1}{4} \\ -4x + 2 & if\ \frac{1}{4} \leq x \leq \frac{1}{2} \\ 0 & if\ \frac{1}{2} \leq x \leq 1 \end{cases}$$

$$\mu_3(x) = \begin{cases} 0 & if\ 0 \le x \le \frac{1}{4} \\ \frac{4x-1}{3} & if\ \frac{1}{4} \le x \le 1 \end{cases}$$

Define $\tau : I^X \rightarrow I$ by $\tau(\mu) = \begin{cases} 1 & if\ \mu = \overline{0}\ or\ \overline{1} \\ \frac{1}{4} & if\ \mu = \mu_1, \mu_2, \mu_1 \wedge \mu_2 \\ 0 & otherwise. \end{cases}$

Now τ is a fuzzy topology on X. Also, the fuzzy sets $\mu_1{}^c$ $and\ \mu_2{}^c$ are $\frac{1}{4}$ **- fuzzy** $\boldsymbol{\omega}$ closed sets.

Proposition 3.3: Every r-fuzzy closed set is r-fuzzy ω closed set but the converse is not necessarily true.

Proof: Let α be any r-fuzzy closed set and μ be a fuzzy r-semiopen set such that $\alpha \le \mu$ in a fuzzy topological space (X, τ). By [3], α =cl(α,r), so that α is a r-fuzzy ω closed set.

Proposition 3.4: In a fuzzy topological space (X, τ), r-fuzzy ω closed set α is r-fuzzy closed set provided α is a fuzzy r-semiopen set.

Proof: Given α is both r-fuzzy ω closed set and fuzzy r-semiopen set in a fuzzy topological space (X, τ). By hypothesis, $cl(\alpha, r) \le \alpha$. Always, $cl(\alpha, r) \ge \alpha$ and so $cl(\alpha, r) = \alpha$ which shows α is r-fuzzy closed set.

Proposition 3.5: Every r-fuzzy ω closed set is r- generalized fuzzy closed set and the converse does not hold as in Example 3.7.

Proof: Let α be any r-fuzzy ω closed set in a fuzzy topological space (X, τ). Let $\alpha \le \mu$ where μ is a fuzzy r-open set in (X, τ). By [7], μ is a fuzzy r-semiopen set. By hypothesis, $l(\alpha, r) \le \mu$. Then, α is r- generalized fuzzy closed set.

Proposition 3.6: If α is r-fuzzy ω closed set in a fuzzy topological space (X, τ) and suppose $\alpha \le \mu \le cl(\alpha, r),\ \mu \in I^X$, then μ is r-fuzzy ω closed set.

Proof: Let ρ be any fuzzy r-semiopen set such that $\mu \le \rho$. By hypothesis and by definition 3.1, $cl(\alpha, r) \le \rho$. Again by hypothesis, $\mu \le cl(\alpha, r)$. By [3], $cl(\mu, r) \le cl(cl(\alpha, r), r) = cl(\alpha, r)$. Now $l(\mu, r) \le cl(\alpha, r) \le \rho \Longrightarrow cl(\mu, r) \le \rho$, so that μ is r-fuzzy ω closed set.

Theorem 3.7: For any $r \in I_0$, α is r-fuzzy ω closed set in a fuzzy topological space (X, τ) if and only if $\alpha \bar{q} \mu \Longrightarrow cl(\alpha, r) \bar{q} \mu$ where μ is fuzzy r-semiclosed set.

Proof: Suppose that α is r-fuzzy ω closed set in a fuzzy topological space (X, τ), $r \in I_0$ and μ is fuzzy r-semiclosed set such that $\alpha \bar{q} \mu$. Then, $\alpha \le \overline{1} - \mu$. Since complement of fuzzy r-semiclosed set is fuzzy r-semiopen set, $\overline{1}-\mu$ is fuzzy r-semiopen. By definition 3.1, cl$(\alpha,r) \leq \overline{1}-\mu$ which implies $cl(\alpha,r)\overline{q}\mu$.

Conversely, assume that the given condition holds. Let μ be any fuzzy r-semiopen set such that $\alpha \leq \mu$.Then,$\alpha\,\overline{q}(\overline{1}-\mu)$. By hypothesis, $cl(\alpha,r)\overline{q}(\overline{1}-\mu)$. Again, $cl(\alpha,r) \leq \mu$ which says by definition 3.1, α is r-fuzzy ω closed set.

Theorem 3.8: In a fuzzy topological space (X, τ), if x_p is a fuzzy point in X and α is any r-fuzzy ω closed set such that $x_p q cl(\alpha,r)$, then$cl(x_p)q\alpha$.

Proof: On contrary, assume that x_p is a fuzzy point in X and α is any r-fuzzy ω closed set in a fuzzy topological space (X, τ) such that $cl(x_p)\overline{q}\ \alpha$. Then, $cl(x_p) \leq (\overline{1}-\alpha)$ or $\leq (\overline{1}-cl(x_p))$. Since $cl(x_p)$ is a r-fuzzy closed set and by [..], $cl(x_p)$ is a fuzzy r-semiclosed set. Since α is r-fuzzy ω closed set by definition 3.1,

$\boldsymbol{cl(\alpha,r)} \leq (\boldsymbol{\overline{1}-cl(x_p)}) \leq) \leq (\boldsymbol{\overline{1}-x_p})$ **. Now,** $\boldsymbol{cl(\alpha,r)\overline{q}\ \ x_p}$

a contradiction. Hence the Theorem**.**

4. R-fuzzy ω-Continuous Functions and R-fuzzy ω-Homeomorphisms

Definition 4.1: Let (X, τ) and (Y, ρ) be any two fuzzy topological spaces. A mapping f: (X, τ) → (Y, ρ) is called

(1) r-fuzzy ω-continuous if $f^{-1}(\mu)$ is a r-fuzzy ω open set in X for any $\mu \in I^Y, r \in I^0$ such that $\rho(\mu) \geq r$.

(2) r-fuzzy ω-irresolute if $f^{-1}(\mu)$ is a r-fuzzy ω open set in X for any r-fuzzy ω open set μ in Y.

(3) r-fuzzy ω-open (resp. r-fuzzy ω-closed) if f(μ) is r-fuzzy ω open (resp. r-fuzzy ω closed) set in Y for any $\mu \in I^X, r \in I^0$ such that $\tau(\mu) \geq r$ (resp. $\tau(\overline{1}-\ \mu) \geq r$).

Proposition 4.2: A mapping f: (X, τ) → (Y, ρ) is r-fuzzy ω-continuous iff pre image of any r-fuzzy closed set of Y is
r-fuzzy ω-closed set of X.

Proof: Let μ be any r-closed set of Y. Then $(\overline{1}-\ \mu)$ is r-fuzzy open set of Y. By hypothesis, $f^{-1}(\overline{1}-\ \mu) =\ \overline{1}-f^{-1}\ (\mu)$ is r-fuzzy ω open set of X. Now, $f^{-1}\ (\mu)$ is r-fuzzy ω-closed set of X.

Conversely, let $\alpha \in I^Y$ be such that $\rho(\alpha) \geq r$ for any $r \in I_0$. Then,

$\rho(\alpha) = \rho(\bar{1} - (\bar{1} - \alpha)) \geq r$ which gives $(\bar{1} - \alpha)$ is a r-fuzzy closed set of Y. By hypothesis, $f^{-1}(\bar{1} - \alpha) = \bar{1} - f^{-1}(\alpha)$ is r-fuzzy ω-closed set of X in turns $f^{-1}(\alpha)$ is r-fuzzy ω-open set of X. Hence the result.

Proposition 4.3: Every fuzzy r-continuous function is r-fuzzy ω-continuous map.

Proof: It follows from the Proposition 3.3.

Proposition 4.4: Let f: (X, τ) → (Y, ρ) is a r-fuzzy ω-continuous function. Then the following statements hold.

i) for any fuzzy point x_p of X and for any r-fuzzy open set α of Y such that $f(x_p) \in \alpha$, there exists a r-fuzzy ω open set μ of X such that $f(\mu) \leq \alpha$.

ii) for any r-fuzzy open set α of Y and for any fuzzy point x_p of X such that $f(x_p) q\, \alpha$, there exists a r-fuzzy ω open set μ of X such that $x_p q \mu$ and $f(\mu) \leq \alpha$.

iii) for any r-fuzzy continuous map g: (Y, ρ) → (Z, η), the composition mapping gof: (X, τ) → (Z, η) is r-fuzzy ω-continuous map.

Proof: i) Let x_p be any fuzzy point of X and α be any r-fuzzy open set of Y such that $f(x_p) \in \alpha$. By hypothesis, $f^{-1}(\alpha)$ is r-fuzzy ω-open set of X such that $x_p \in f^{-1}(\alpha)$.

By taking $\mu = f^{-1}(\alpha),\ it\ leads\ to\ f(\mu) = f\left(f^{-1}(\alpha)\right) \leq \alpha$.

ii) Let x_p be any fuzzy point of X and α be any r-fuzzy open set of Y such that $f(x_p) q\, \alpha$. By hypothesis, $f^{-1}(\alpha)$ is r-fuzzy ω-open set of X such that $x_p q\, f^{-1}(\alpha)$ By taking $\mu = f^{-1}(\alpha),\ f(\mu) \leq \alpha$.

iii)Let α be any r-fuzzy open set of Z. By hypothesis, $g^{-1}(\alpha)$ is r-fuzzy open set of Y.

By hypothesis, $f^{-1}\left(g^{-1}(\alpha)\right) = (gof)^{-1}(\alpha)$ is r-fuzzy ω-open set of X. Hence (iii) holds.

Proposition 4.5: Let f: (X, τ) → (Y, ρ) is a r-fuzzy ω-irresolute mapping. Then the following statements hold.

(1) i)for any fuzzy point x_p of X and for any r-fuzzy ω open set α of Y such that $f(x_p) \in \alpha$, there exists a r-fuzzy ω open set μ of X such that $f(\mu) \leq \alpha$.

ii)for any r-fuzzy ω open set α of Y and for any fuzzy point x_p of X such that $f(x_p) q\ \alpha$, there exists a r-fuzzy ω open set μ of X such that $x_p q\mu$ and $f(\mu) \leq \alpha$.

iii)for any r-fuzzy ω-irresolute map g: (Y, ρ) → (Z, η), the composition mapping gof: (X, τ) → (Z, η) is r-fuzzy ω-irresolute function.

Proof: i) Let x_p be any fuzzy point of X and α be any r-fuzzy ω open set of Y such that $f(x_p) \in \alpha$. By hypothesis, $f^{-1}(\alpha)$ is r-fuzzy ω-open set of X such that $x_p \in f^{-1}(\alpha)$. By taking $\mu = f^{-1}(\alpha)$, *it leads to* $f(\mu) = f\left(f^{-1}(\alpha)\right) \leq \alpha$.

ii) Let x_p be any fuzzy point of X and α be any r-fuzzy ω open set of Y such that $f(x_p)q\ \alpha$. By hypothesis, $f^{-1}(\alpha)$ is r-fuzzy ω-open set of X such that $x_p q\ f^{-1}(\alpha)$ By taking $\mu = f^{-1}(\alpha)$, $f(\mu) \leq \alpha$.

iii)Let α be any r-fuzzy ω open set of Z. By hypothesis, $g^{-1}(\alpha)$ is r-fuzzy open set of Y. By hypothesis, $f^{-1}\left(g^{-1}(\alpha)\right) = (gof\ )^{-1}(\alpha)$ is r-fuzzy ω-open set of X. Hence iii) holds.

Theorem 4.6: A mapping f: (X, τ) $\rightarrow$ (Y, ρ) is r-fuzzy ω-closed if and only if for each $\lambda \in I^Y$ and for each r-fuzzy open set μ of X such that $f^{-1}(\lambda) \leq \mu$, there exists a r fuzzy ω open set η of Y such that $\lambda \leq \eta$ and $f^{-1}(\eta) \leq \mu$.

Proof: Let $\lambda \in I^Y$ and μ be any r-fuzzy open set of X such that $f^{-1}(\lambda) \leq \mu$. Then, $\overline{1}$- μ is r-fuzzy closed set of X. By hypothesis, f($\overline{1}$- μ) is r-fuzzy ω-closed set in Y. Then, $\overline{1} -$ f($\overline{1}$- μ) is r-fuzzy ω-open set in Y. By choosing $\eta = \overline{1} -$ f($\overline{1}$- μ) , η is a r-fuzzy ω-open set in Y such that $\lambda \leq \eta$ and $f^{-1}(\eta) \leq \mu$.

Conversely, let α be any r-fuzzy closed set

Clearly f(α) $\in I^Y$ and $f^{-1}\left(\overline{1} - f(\alpha)\right) = \overline{1} - f^{-1}\left(f(\alpha)\right) \leq \overline{1} - \alpha$. Now $\overline{1} - \alpha$ is r-fuzzy open set in X such that $f^{-1}\left(\overline{1} - f(\alpha)\right) \leq \overline{1} - \alpha$. By hypothesis, there exists r fuzzy ω open set η of Y such that $\left(\overline{1} - f(\alpha)\right) \leq \eta$ and $f^{-1}(\eta) \leq \overline{1} - \alpha$ *and hence* $\alpha \leq \overline{1} - f^{-1}(\eta)$. Now, $\overline{1} - \eta \leq f(\alpha) \leq f(\overline{1} - f^{-1}(\eta)) \leq \overline{1} - \eta$ and so $f(\alpha) = \overline{1} - \eta$. Then $f(\alpha)$ is r-fuzzy ω closed set in Y.

5. r-fuzzy ω-Homeomorphsim:

Definition 5.1: A bijective mapping f: (X, τ) $\rightarrow$ (Y, ρ) is called r-fuzzy ω-homeomorphsim if f and f^{-1} are r-fuzzy ω-continuous.

Theorem 5.2: For any bijective map f: (X, τ) $\rightarrow$ (Y, ρ), the following are equivalent.

(1) f is r-fuzzy ω-homeomorphsim

(2) f is r-fuzzy ω-continuous and r-fuzzy ω-open map

(3) f is r-fuzzy ω-continuous and r-fuzzy ω-closed map.

Proof(1)$\Longrightarrow$ (2)Let f be r-fuzzy ω-homeomorphsim. By definition 5.1, f and f^{-1} are r-fuzzy ω-continuous. It is enough to prove that f is r-fuzzy ω-open map. Let α be any r-fuzzy open set in X. Since f^{-1}: (Y, ρ) → (X, τ) is r-fuzzy ω-continuous, $(f^{-1})^{-1}(\alpha) = f(\alpha)$ is r-fuzzy ω open set in Y. So f is r-fuzzy ω-open map.

(2) $\Longrightarrow$ (3) It is enough to prove that f is r-fuzzy ω-closed map. Let α be any r-fuzzy closed set in X. Then, $\overline{1} - \alpha$ is r-fuzzy open set in X. By hypothesis, f($\overline{1} - \alpha$) is r-fuzzy ω open set in Y and so $\overline{1} - f(\alpha)$ is r-fuzzy ω open set in Y. Hence $f(\alpha)$ is r-fuzzy ω closed set in Y. So,f is r-fuzzy ω-closed map.

(3)$\Longrightarrow$(1) It is enough to prove that f^{-1}: (Y, ρ) → (X, τ) is r-fuzzy ω-continuous. Let α be any r-fuzzy open set in X. Then $\overline{1} - \alpha$ is r-fuzzy closed set in X.

By hypothesis, f($\overline{1} - \alpha$) = $\overline{1} - f(\alpha)$ is r-fuzzy ω closed set in Y. Now, $f(\alpha)$ = $(f^{-1})^{-1}(\alpha)$ is r-fuzzy ω open set in Y. So, f^{-1} is r-fuzzy ω-continuous.

CONCLUSION

In this paper, we have introduced the notion of r-fuzzy ω- closed sets in the fuzzy topological space in the sense of Sostak A.P which is a generalization of Chang's fuzzy topology. By using the class of r-fuzzy ω- closed sets, some functions have been defined and they are used in developing the notion of r-fuzzy ω-homeomorphism. The above results can be extended to fuzzy soft topological space and intuitionist fuzzy topological spaces.